Spacecraft Mass Property Identification with Torque-Generating Control

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Previous studies indicated that an applied force was necessary to perform in-flight identification of the mass and center of mass of a spacecraft. This paper shows that the mass and center of mass of a rigid spacecraft can be determined using only torque-producing actuators such as control-moment gyros or reaction wheels, and commonly available sensors, e.g., rate gyros and accelerometers. A space-station application is presented.

Introduction

COMPLEX spacecraft with evolving configurations such as the NASA Space Station or those carrying diverse payloads such as the Space Shuttle or Orbital Maneuvering Vehicle must be controlled over a wide range of mass properties. An autopilot for spacecraft reaction control with a capability to adapt to widely varying spacecraft center of mass, inertia matrix, and mass has been developed[1,2] and successfully tested on the Space Shuttle. The approach taken in this autopilot has been extended to spacecraft controlled by torque-producing devices such as control moment gyros, or a mixture of torque-producing and reaction control devices.[3]

Both autopilots can adapt to a wide range of spacecraft mass properties if such mass properties are known. Moreover, although both systems can tolerate significant errors in the available knowledge of mass properties, their efficiency is strongly correlated with accurate estimation of mass properties.

Currently, spacecraft such as the Space Shuttle require careful estimation of component mass properties as well as prediction and/or measurement of consumables mass. Such a process is tedious at best and susceptible to a number of error sources. For a vehicle such as the Orbital Maneuvering Vehicle, which is to operate with many different payloads, not all of which have known mass properties (one mission is to collect and de-orbit debris), it is not always possible to accurately predict vehicle plus payload mass properties. Efficiency of operation will increase with increased accuracy of mass property estimates.

The preceding considerations have led to the development of an algorithm for the in-flight determination of vehicle mass properties applicable to vehicles using reaction control.[4,5] Intuitively, it is natural to think that an unbalanced force must be applied to the vehicle to determine vehicle mass and center of mass. For certain spacecraft such as the Space Station, using reaction-control jets for mass property identification may be undesirable for three reasons. First, the jets consume propellant, which must be brought up from Earth. Second, operation of the jets produces jerks and high acceleration levels that may disturb the vehicle structure or onboard experiments. Third, use of the jets produces exhaust contaminants that can interfere with onboard experiments. Use of pure torque devices such as control moment gyros or reaction wheels obviates these problems. Such devices are usually powered by electricity available from solar cells or onboard generation, operate more smoothly than reaction control jets, and produce no exhaust. It becomes, then, highly desirable to develop a mass property identification scheme that can use such devices to excite vehicle motion, rather than jets.

The remainder of this paper describes an algorithm for spacecraft mass property identification using torque-producing control. Separate sections are devoted to the identification of spacecraft inertia matrix, center of mass, and mass. Finally, example applications for the Space Station are presented.

Identification of the Inertia Matrix

Identification of the inertia matrix can be formulated as a Kalman filter as follows. Let

$$x^T = (Ixx^{-1}\ Iyy^{-1}\ Izz^{-1}\ Ixy^{-1}\ Ixz^{-1}\ Iyz^{-1}) \tag{1}$$

be the vector to be estimated, whose elements are the six unique elements of the inverse inertia matrix (e.g., I_{xx}^{-1} is the element of the inverse inertia matrix corresponding to I_{xx}). Assuming the mass properties to be constant between measurements, the propagation equation is simply

$$x_{\text{new}} = x_{\text{old}} \tag{2}$$

Incorporation of measurements is performed via

$$x_{\text{new}} = x_{\text{old}} + K(\Delta w_m - \Delta w_p) \tag{3}$$

where K is a 6×3 gain matrix, Δw_m the measured angular rate change over some interval, and Δw_p the predicted rate change over the same interval.

The predicted rate change is given by

$$\Delta w_p = H\, x_{\text{old}} \tag{4}$$

where

$$H = \begin{bmatrix} S1 & 0 & 0 & S2 & S3 & 0 \\ 0 & S2 & 0 & S1 & 0 & S3 \\ 0 & 0 & S3 & 0 & S1 & S2 \end{bmatrix} \tag{5}$$

and $S1$, $S2$, $S3$, are roll, pitch, and yaw impulses applied to the spacecraft.

The Space Station will use double-gimballed control-moment gyros (CMG) for control. For simplicity and to minimize

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motion of the station, it was decided to perform the estimation by moving one gimbal at a time and comparing the predicted and measured rate change due to that gimbal motion. A parallel configuration of CMG's has been proposed for Space Station application.[6] The CMG's are oriented with parallel outer gimbals rotating about the roll axis and parallel inner gimbals rotating about the yaw axis. For a CMG mounted in this manner, the torque due to rotation of the inner gimbal of the ith CMG (all other gimbals fixed) in vehicle body roll, pitch, and yaw coordinates is

$$T_i = \begin{pmatrix} h\,\dot{\theta}_i \cos\theta_i \\ -h\,\dot{\theta}_i \sin\theta_i \cos\phi_i \\ -h\,\dot{\theta}_i \sin\theta_i \sin\phi_i \end{pmatrix} \tag{6}$$

where h is the angular momentum magnitude of the CMG rotor, θ_i the inner gimbal angle of the ith CMG, and ϕ_i the outer gimbal angle of the ith CMG. The corresponding torque due to rotation of the outer gimbal of CMG i is

$$T_i = \begin{pmatrix} 0 \\ -h\,\dot{\phi}_i \cos\theta_i \sin\phi_i \\ h\,\dot{\phi}_i \cos\theta_i \cos\phi_i \end{pmatrix} \tag{7}$$

The torque impulse applied by rotating an inner gimbal at constant rate $\dot{\theta}$ for t_i s is given by integration of Eq. (6).

Noting that

$$\begin{aligned} &\int_0^{t_i} h\,\dot{\theta}\cos\theta\,\mathrm{d}t \\ &= \int_0^{t_i} h\cos\theta\,\frac{\mathrm{d}\theta}{\mathrm{d}t}\,\mathrm{d}t \\ &= \int_{\theta_0}^{\theta_f} h\cos\theta\,\mathrm{d}\theta \end{aligned} \tag{8}$$

one obtains

$$S = \begin{bmatrix} h\,(\sin\theta_f - \sin\theta_0) \\ h\cos\phi\,(\cos\theta_f - \cos\theta_0) \\ h\sin\phi\,(\cos\theta_f - \cos\theta_0) \end{bmatrix} \tag{9}$$

where θ_f = inner gimbal angle at the end of interval $[0, t_i]$ and θ_0 = inner gimbal angle at the start of interval $[0, t_i]$.

Similarly, for an outer gimbal displacement

$$S = \begin{bmatrix} 0 \\ h\cos\theta\,(\cos\phi_f - \cos\phi_0) \\ h\cos\theta\,(\sin\phi_f - \sin\phi_0) \end{bmatrix} \tag{10}$$

where ϕ_f = outer gimbal angle at the end of interval $[0, t_i]$ and ϕ_0 = outer gimbal angle at the beginning of interval $[0, t_i]$.

The gain matrix K can be expressed

$$K = P\,H^T M^{-1} \tag{11}$$

where M is given by

$$M = H\,P\,H^T + R \tag{12}$$

Here, R is the measurement noise covariance and P the error covariance. After each measurement incorporation, the error covariance is updated via

$$P_{\text{new}} = P_{\text{old}} - P_{\text{old}}\,H^T M^{-1} H P_{\text{old}} \tag{13}$$

Identification of the Spacecraft Center of Mass

The output of an accelerometer placed on the spacecraft includes terms arising from both translation and rotation:

$$\boldsymbol{a} = \boldsymbol{a}_{\text{cm}} + \dot{\boldsymbol{w}} \times \boldsymbol{r} + \boldsymbol{w} \times (\boldsymbol{w} \times \boldsymbol{r}) + 2\boldsymbol{w} \times \dot{\boldsymbol{r}} \tag{14}$$

where

$\boldsymbol{a}$ = sensed acceleration
$\boldsymbol{a}_{\text{cm}}$ = acceleration of spacecraft center of mass
$\boldsymbol{w}$ = spacecraft angular rate
$\boldsymbol{r}$ = displacement of accelerometer from center of mass

When no unbalanced forces are applied, the center of mass does not accelerate, hence $\boldsymbol{a}_{\text{cm}} = 0$. For a rigid spacecraft, the displacement of the accelerometer from the center of mass (in body coordinates) is fixed, hence $\dot{\boldsymbol{r}} = 0$. Equation (14) then reduces to

$$\boldsymbol{a} = \dot{\boldsymbol{w}} \times \boldsymbol{r} + \boldsymbol{w} \times (\boldsymbol{w} \times \boldsymbol{r}) \tag{15}$$

or

$$\boldsymbol{a} = (H_1 + H_2)\boldsymbol{r} \tag{16}$$

where

$$H_1 = \begin{bmatrix} 0 & -\dot{w}_z & \dot{w}_y \\ \dot{w}_z & 0 & -\dot{w}_x \\ -\dot{w}_y & \dot{w}_x & 0 \end{bmatrix} \tag{17}$$

$$H_2 = \begin{bmatrix} -(w_y^2 + w_z^2) & w_x w_y & w_x w_z \\ w_x w_y & -(w_x^2 + w_z^2) & w_y w_z \\ w_x w_z & w_y w_z & -(w_x^2 + w_y^2) \end{bmatrix} \tag{18}$$

$$\boldsymbol{r} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} \tag{19}$$

Note that H_1 and H_2 are singular. A Kalman filter has been developed that extracts $\boldsymbol{r}$ from measurements of $\boldsymbol{w}$ and $\boldsymbol{a}$. This filter takes the form

$$r = r_{\text{old}} + K(a - Hr_{\text{old}}) \tag{20}$$

where the gain and covariance are calculated as in Eqs. (11-13). Measurements of $\boldsymbol{w}$ and $\dot{\boldsymbol{w}}$ must be accurate for the filter to perform well, as H in Eq. (20) is a function of those measurements. High quality rate gyros are available for $\boldsymbol{w}$, but the performance of angular accelerometers to date is not adequate for accurate mass property estimation. Hence, estimation of the center of mass will not be performed while torque is being applied, but by comparison of rate changes across an impulse.

For a vehicle rotating at a constant angular rate, H is given by H_2 [Eq. (18)]. The inertia estimator can operate without knowledge of the center-of-mass location and can be run prior to initiating center-of-mass identification. Doing so provides knowledge of the inverse inertia, so that CMG's can be used to provide a desired angular rate for the estimator.

Identification of Spacecraft Mass

The mass of the Space Station can be determined without the application of external force by the center-of-mass estimator. The station center of mass is estimated twice, the difference between measurements being that a component of known mass is moved. If the mass of that component is a sufficiently large fraction of the total spacecraft mass, and the displacement between measurements is sufficiently great, the center of mass will be measurably altered.

The initial center-of-mass measurement R can be expressed as

$$R = \frac{M_c r_c + M_s r_s}{M_c + M_s} \tag{21}$$

where

M_c = mass of movable component (assumed known)
M_s = mass of all other Space Station components (to be found)
r_c = location of component mass center in a frame fixed to the Space Station
r_s = location of the composite center of mass of all other Space Station components in the same frame as r_c (unknown)

After the component is displaced, the new center-of-mass location is

$$R' = \frac{M_c r_c' + M_s r_s}{M_c + M_s} \tag{22}$$

Clearly, Eqs. (21) and (22) can be solved for M_s (note that r_s drops out):

$$M_s = \frac{M_c(r_c - r_c')}{(R - R')\cdot(r_c - r_c')}[r_c - r_c' - (R - R')] \tag{23}$$

Hence, if M_c, r_c, and r_c' are known, and R and R' are estimated, the mass of the Space Station can be determined without the application of an external force.

Rather than moving a mass aboard the Space Station, one could perform mass identification as part of docking operations. Prior to docking, the Space Station center of mass R_s is determined. After a spacecraft with known mass m_0 docks with the Space Station, the new center of mass is

$$R_s' = \frac{R_s m_s + R_0 m_0}{m_s + m_0} \tag{24}$$

from which the Space Station mass is derived:

$$m_s = \frac{m_0[R_0 - R_s']\cdot[R_s' - R_s]}{[R_s' - R_s]^2} \tag{25}$$

Clearly, a similar expression applies if a spacecraft separates from the Space Station, or if a large mass of fuel is consumed.

Equation (22) or (25) can then be used as a measurement equation for a Kalman filter formulation of the mass estimation over several displacements of the reference components.

Input Selection

Inputs for the mass property estimators could consist of any known torques applied to the Space Station. There is no guarantee, however, that normal control activity will provide a set of inputs sufficiently rich to determine all mass properties. Rather, the estimator may be used as a check of current mass property estimates during normal control activity: a large residual indicates significant errors in the current mass property estimates. To insure sufficient richness of inputs for mass property identification, a scheme analogous to that of Ref. 5 is proposed.

Each estimator is formulated assuming a sequence of rotations of single CMG gimbals. The change in covariance at each step is [from Eq. (13)]

$$\Delta P = P_{\text{old}} H^T M^{-1} H P_{\text{old}} \tag{26}$$

Convergence is most rapid if at each step the input is selected to maximize this change in covariance. To maximize the change at each step, Eq. (26) is computed for each gimbal of each CMG, and that gimbal that maximizes the trace of ΔP is selected.

It will be seen that vehicle motion is small during the estimation process. This is generally true as the vehicle needs only to be moved sufficiently for the sensors to provide good measurements of the spacecraft motion. If at any point in the estimation process the vehicle accumulates unacceptable rates, the estimation process could be stopped and rates damped before proceeding further. The ability to damp rates, however, assumes good initial knowledge of the vehicle inertias.

Space Station Examples

The estimation algorithms have been combined into a single estimation package and tested with simulations of a rigid model of the NASA Space Station. This package has been combined with the CMG steering law described in Ref. 3. The package first estimates the Space Station inertia matrix, and then the location of the center of mass. For mass estimation, a proposed mobile version of the Shuttle remote manipulator is used. This system, weighing approximately 1000 lb, will have known mass, and can be easily moved to several locations on the Space Station. Successive center-of-mass estimations are performed with the manipulator in different locations. At present, the motion is assumed to occur instantaneously. In practice, normal control operations would resume while the manipulator was being moved. The measurements of the center of mass are then used to estimate the Space Station mass.

At each iteration of the filters the predicted angular rate of the spacecraft is computed assuming a CMG gimbal has been rotated a fixed amount. The predicted change in the error covariance matrix is then computed for each gimbal assuming only one gimbal is to be rotated at a time. The gimbal that produces the largest change in the trace of the error covariance matrix is then turned on for 5 s at the peak gimbal of 5 deg/s, producing an angular rate of order 10^{-2} deg/s on the station. The advantage of this approach is that the estimates converge rapidly; the disadvantage is that it is necessary to suspend attitude control functions while the mass properties are being estimated. However, this is not likely to be a serious difficulty, as the process takes only a few minutes, and disturbance

Table 1 Space station mass properties

Inertia matrix with respect to center of mass, kg m^2 × 10^7			Center of gravity, m	Mass, kg
2.3320	−0.0778	0.1206	−7.553	
−0.0778	1.8459	0.1246	1.377	53630.9
0.1206	0.1246	1.4696	1.074	

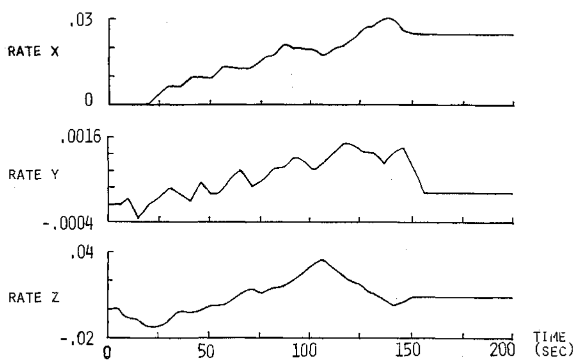

Fig. 1 Spacecraft angular rate, deg/s: estimation inputs.

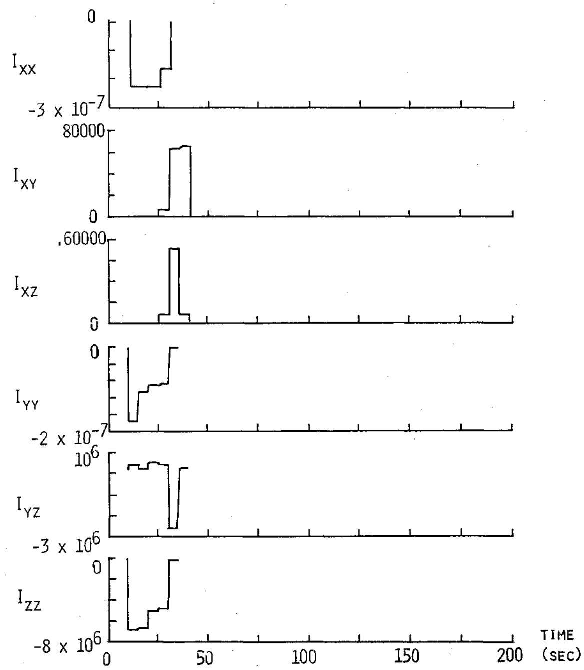

Fig. 2 Percent error in inertia estimate: estimation inputs.

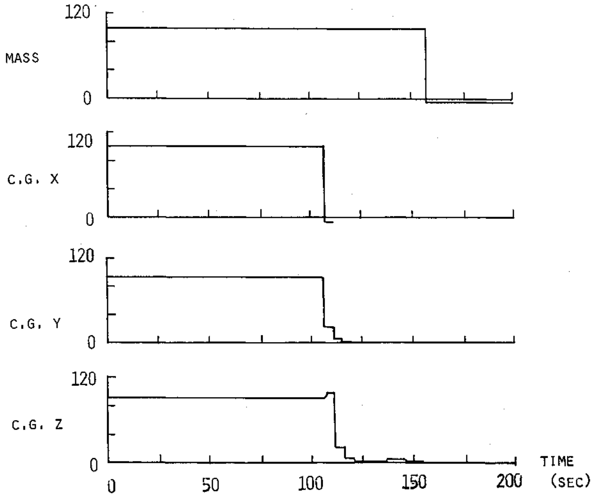

Fig. 3 Percent error in center of gravity and mass estimate estimation inputs.

torques will be small. In addition, a convergence criterion has been added that terminates the estimation of the inertia matrix and the center of mass after a specified number of consecutive rate residuals $|\Delta w_m - \Delta w_p|$ are below a certain threshold.

Results from two simulations are presented that demonstrate how rapidly the algorithm converges to the actual values of the inertia properties. In the first example, the inertia properties are assumed to be completely unknown, and in the second, a spacecraft of substantial and known mass is "docked" with the Space Station.

In the first simulation, the inertia estimator is initialized with the values $I_{zz} = I_{yy} = I_{zz} = 0.01$ kg m^2 and $I_{xy} = I_{xz} = I_{yz} = 0.0$ kg m^2. The exact values of the vehicle mass properties are shown in Table 1. Time histories are shown in Fig. 1 for the attitude rates and in Fig. 2 for the error in estimate of the elements of the inertia matrix. With one exception (I_{yz}), the algorithm has identified the elements of the inertia matrix to within a fraction of a percent after 90 s, as indicated on the plots. The initial values of the center-of-mass estimate are $r_x = r_y = r_z = 0.1$ m. The error histories of the center of mass and mass estimates are shown in Fig. 3. The initial estimation of the center of mass requires about 40 s, and the error is approximately 2%. The second estimation requires 15 s and yields a final error of less than 1.3% in each component. The spacecraft mass is estimated once after the second estimate of the center of mass is obtained and yields an error of about 3.4%. Repeated trials improved this result considerably.

In the second simulation, the inertia estimator is initialized with the exact values of the inertia matrix given in Table 1, and the autopilot commands a sequence of rate changes. Figure 4 shows time histories of the Space Station attitude rate in body coordinates. First, a rate of 0.01 deg/s is commanded about the roll axis and then removed. Following this, a rate of 0.005 deg/s is commanded and removed from the pitch axis and then the yaw axis. At 140 s into the simulation, a model of the Space Shuttle Orbiter is docked with the space station. The

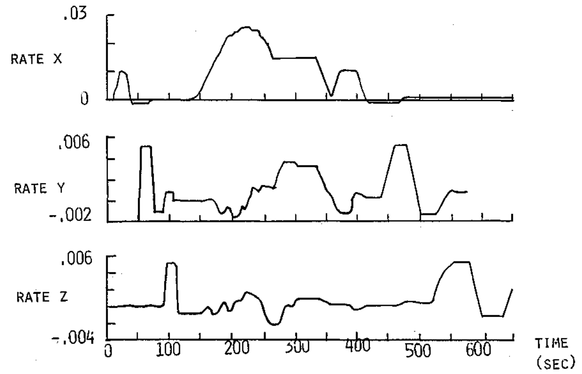

Fig. 4 Spacecraft angular rate, deg/s: Shuttle docking.

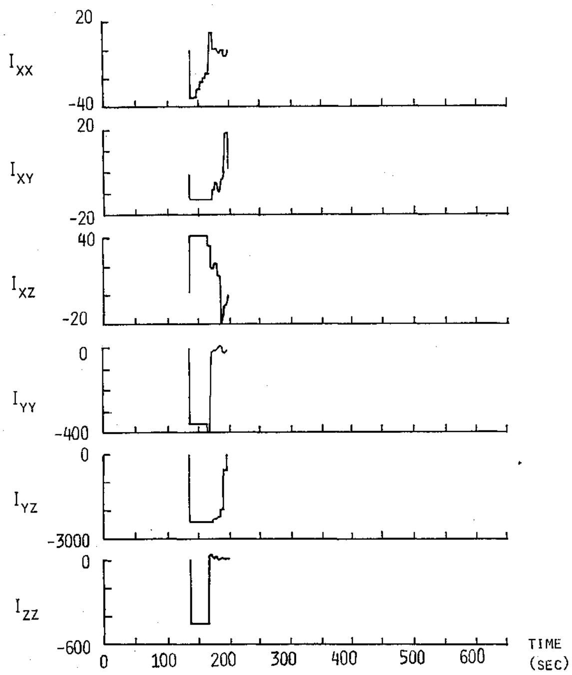

Fig. 5 Percent error in inertia estimate: Shuttle docking.

Table 2 Space Shuttle mass properties

Inertia matrix with respect to center of mass, kg m^2 × 10^6			Center of gravity, m	Mass, kg
6.4423	0.1831	1.9740	−27.91	
0.1831	49.4407	0.0270	0.020	113721.9
1.9740	0.0270	51.5927	−9.533	

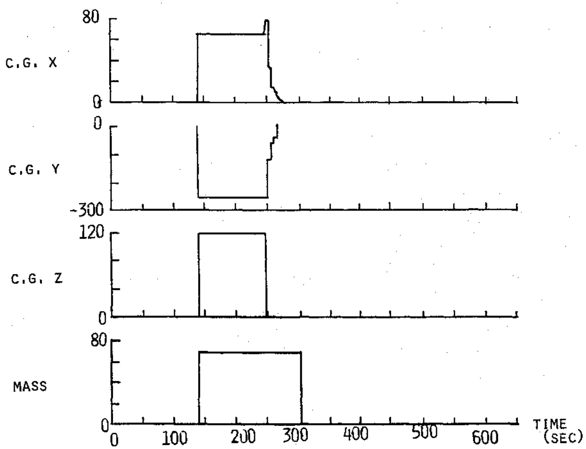

Fig. 6 Percent error in center of gravity and mass estimate: Shuttle docking.

inertia properties of the Orbiter are given in Table 2. The inertia matrix is given with respect to the center of mass. After the docking transient is damped, the estimation process is repeated. Figure 5 shows the percent error in the estimate of the inertia matrix. The diagonal elements of the inertia matrix have been identified to within a fraction of a percent. Two of the off-diagonal terms are accurate to only about 10%. Greater accuracy could have been obtained by tightening the restriction on the convergence criterion. The estimation of the inertia matrix lasted 100 s. The first estimate of the center of mass required 40 s, and the second estimate required 15 s, yielding an estimate of the center of mass accurate to a fraction of an inch. The final estimate of the mass had an error of approximately 0.2%. After the mass properties have been estimated, the sequence of roll, pitch, and yaw rate commands are repeated. The autopilot still works effectively, but the response is slower due to the substantial increase in inertia of the vehicle and corresponding decrease in control authority of the CMG's.

Conclusion

An algorithm has been developed for estimation of spacecraft mass properties using only torque-producing actuators. This capability enhances the usefulness of an autopilot capable of adapting to changes in spacecraft mass properties. The use of torque-producing control for estimation prevents fuel usage, disturbances, and contamination problems associated with the use of reaction control for mass property identification.

This study considered only rigid spacecraft. Work is underway to determine the effects of spacecraft flexibility on the estimator.

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